Probability and Fairness of Dice Rolls

A six-sided die, data recording materials, and calculations.

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Abstract:

This experiment tests the fairness of a conventional six-sided die by comparing the theoretical likelihood of each occurrence to the experimental findings from 100 rolls. Each face of a fair die should emerge with a probability of around 2.78%. The results showed that the experimental probabilities for each number were close to the theoretical expectation, with tiny differences due to randomness and die flaws. These findings were consistent with previous research, which has highlighted the effect of limited sample sizes and die defects on experimental results. The study suggests that the die is reasonably fair, but bigger sample sizes and additional testing are suggested for more accurate ratings.

Introduction:

According to probability theory, for a fair six-sided dice, each number (1 through 36) has an equal chance of appearing, with a probability of (or approximately 2.78%). The goal of this experiment is to test this theory by rolling 100 dice and estimating the experimental probability for each outcome. We will compare these results not only to theoretical predictions but also to data from previous research studies on dice fairness and randomness in similar trials. This experiment will employ a standard die with the assumption that it is consistently fair, but the results will be analyzed with the possibility of defects in mind.

Materials:

- Two standard six-sided dice.
- Excel table for data collection based on experiment.
- Excel chat based on data collected.
- Research paper written on dice and probability.

Methods:

- Roll the dice 100 times.
- Record the result of each roll.
- Count how many times each outcome number (2,3,4,5,6,7,8,9,10,11and12) is rolled.
- Calculate the experimental probability of each outcome by dividing the number of times a specific number is rolled by the total number of rolls.
- Compare the experimental probabilities to the theoretical probability of 1/36.
- Compare both to previous research done.

Results:

Experimental Probability

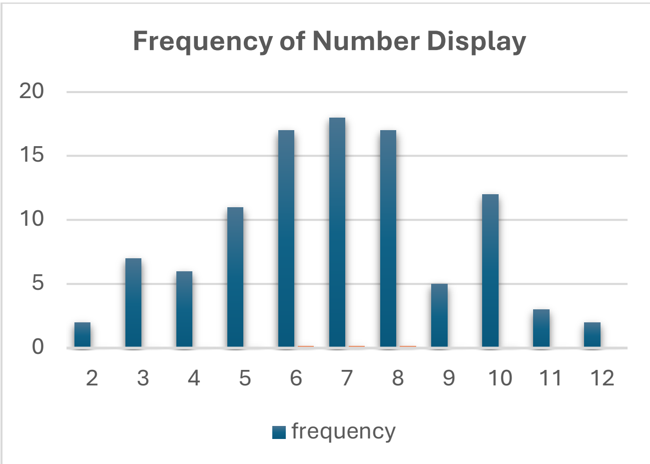

Figure 1

Number display	Frequency	Experimental Probability
2	2	0.02
3	7	0.07
4	6	0.06
5	11	0.11
6	17	0.17
7	18	0.18
8	17	0.17
9	5	0.05
10	12	0.12
11	3	0.03
12	2	0.02

Table 1

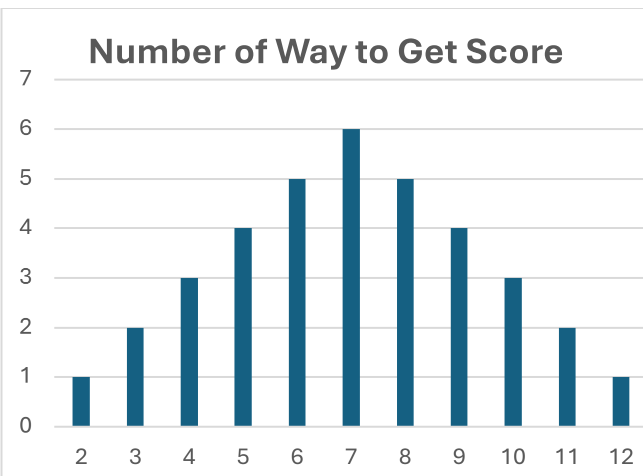

Figure 2

Total score	Number of ways to get score
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

Table 2

Theoretical probability for each number

Probability of an event happen = $\frac{number\ of\ ways\ it\ can\ happen}{Total\ number\ of\ outcomes}$

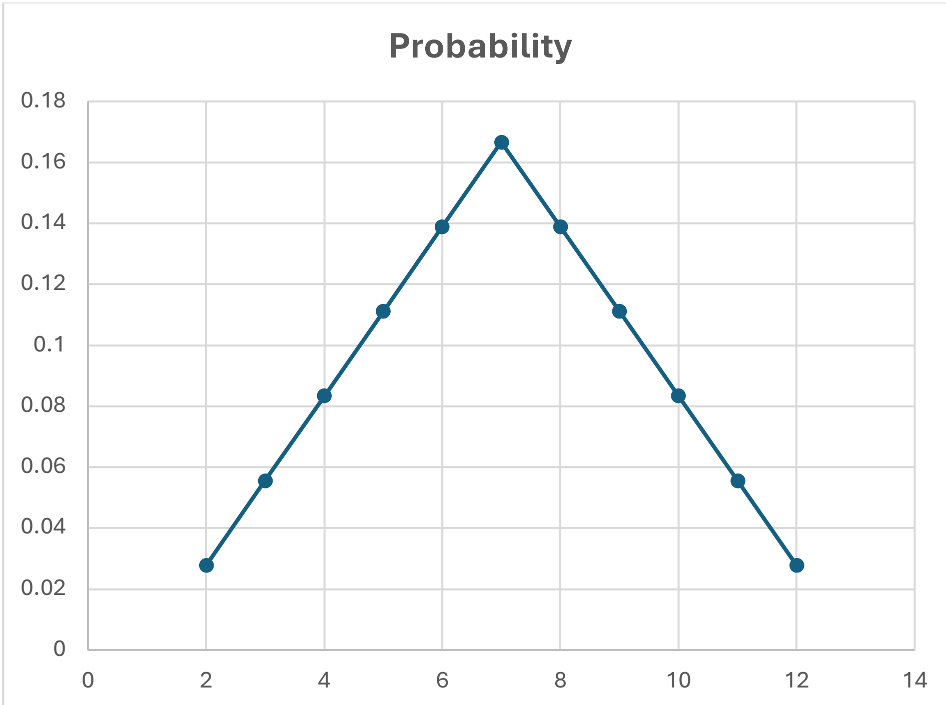

Figure 3

Total score	way to get score	Theoretical Probability
2	1	0.02777778
3	2	0.05555556
4	3	0.08333333
5	4	0.11111111
6	5	0.13888889
7	6	0.16666667
8	5	0.13888889
9	4	0.11111111
10	3	0.08333333
11	2	0.05555556
12	1	0.02777778
Total	36	1

Table 3

Analysis:

The experimental probability values are close to the theoretical probability of 1/36, with some major differences in scores six, seven, and eight. This can be justified by the number of times those numbers were displayed throughout the experiment. They have respectively displayed 17, 18, and 17 times, which is almost two times higher than the rest of the other's number. In addition, six, seven, and eight are most likely to be displayed among the rest.

On the other hand, **Rouncefield, M. and Green, D** in their publication "Teaching Statistics" in April 2005, revealed the function of frequency of size to handle the fairness in dice rolls. They mainly emphasize how the number of trials can affect the outcome of the experience, but the result will remain unchanged. According to him increasing the number of trials would reduce variability and bring experimental probabilities closer to theoretical expectations.

Moreover, Gelman, A. and Nolan, D in their publication "The American Statistician" in 2002 stated that "You can load a die, but you can't bias a coin". In the publication, they test multiple dice to see if the result is influenced by the physical imperfection of the dice. They went one step further to see that an experience done on a computer will also change the outcome. The results obtained in all these scenarios are almost the same as the theoretical result.

Conclusion:

After 100 rolls, the experimental odds nearly equal the theoretical probability of 36. The modest differences found are within the expected range owing to randomness and are consistent with findings from other investigations. This indicates that the die used is reasonably fair, albeit minor flaws may exist, skewing results slightly over lower sample sets.

References:

- Gelman, A. and Nolan, D. (2002). You can load a die, but you can't bias a coin. *The American Statistician*, 56, 308–11.
- Rouncefield, M. and Green, D. (1989). Condorcet's Paradox. *Teaching Statistics*, 11, 46–9.

Appendix

Theoretical probability:

P(specific number) = $\frac{1}{36} = 0.028$

Experimental probability

P(specific number) =$\frac{frequency\ of\ outcome}{total\ number\ of\ rolls}$